

This policy is intended to provide guidance to all staff on how to teach the four operations progressively. It is based upon providing pupils with a high-quality mathematics education and shares the aims of the national curriculum.

The national curriculum for mathematics aims to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The policy sets out a system of progression that staff should follow to ensure a consistent teaching approach and so that year on year children's learning is based upon and builds on previous teaching and learning methods. However, the policy still recognises that pupils may need some methods to be simplified or extended to suit their learning needs.

This policy has been written using the Essex Education Service 'Calculation Guidance 2013/2014' and updated to incorporate the Abacus scheme of work that we use in our school.

Addition Calculation Guidance

Year 1

Pupils should be taught (mental methods):

- Number bonds ('story' of 5, 6, 7, 8, 9 and 10)
- To count on in 1s from a given 2-digit number
- Add two 1-digit numbers
- Add three 1-digit numbers, spotting doubles or pairs to 10
- Count on in 10s from any given 2-digit number
- Add 10 to any given 2-digit number
- Use number facts to add 1-digit numbers to 2-digit numbers e.g. Use $4 + 3$ to work out $24 + 3$, $34 + 3$
- Add by putting the larger number first

Year 2

Pupils should be taught (mental methods):

- Number bonds – know all the pairs of numbers which make all the numbers to 12, and pairs with a total of 20
- Count on in 1s and 10s from any given 2-digit number

- Add two or three 1-digit numbers
- Add a 1-digit number to any 2-digit number using number facts, including bridging multiples of 10 e.g. $45 + 4$ e.g. $38 + 7$
- Add 10 and small multiples of 10 to any given 2-digit number
- Add any pair of 2-digit numbers

Year 3

Pupils should be taught to (mental methods):

- Know pairs with each total to 20 e.g. $2 + 6 = 8$, $12 + 6 = 18$, $7 + 8 = 15$
- Know pairs of multiples of 10 with a total of 100
- Add any two 2-digit numbers by counting on in 10s and 1s or by using partitioning
- Add multiples and near multiples of 10 and 100
- Perform place-value additions e.g. $300 + 8 + 50 = 358$
- Use place value and number facts to add a 1-digit or 2-digit number to a 3-digit number e.g. $104 + 56$ is 160 since $104 + 50 = 154$ and $6 + 4 = 10$ and $676 + 8$ is 684 since $8 = 4 + 4$ and $76 + 4 + 4 = 84$
- Add pairs of 'friendly' 3-digit numbers e.g. $320 + 450$
- Begin to add amounts of money using partitioning

Pupils should be taught to (written methods):

- Use expanded column addition to add two or three 3-digit numbers or three 2-digit numbers
- Begin to use compact column addition to add numbers with 3 digits

Year 4

Children should be taught to (mental methods):

- Add any two 2-digit numbers by partitioning or counting on
- Know by heart/quickly derive number bonds to 100 and to £1
- Add to the next 100, £1 and whole number e.g. $234 + 66 = 300$ e.g. $3.4 + 0.6 = 4$
- Perform place-value additions e.g. $300 + 8 + 50 + 4000 = 4358$
- Add multiples and near multiples of 10, 100 and 1000
- Add £1, 10p, 1p to amounts of money
- Use place value and number facts to add 1-, 2-, 3- and 4-digit numbers where a mental calculation is appropriate e.g. $4004 + 156$ by knowing that $6 + 4 = 10$ and that $4004 + 150 = 4154$ so the total is 4160

Children should be taught to (written methods):

- Use compact column addition to add 3-digit and 4-digit numbers

Year 5

Pupils should be taught to (mental methods):

Know number bonds to 1 and to the next whole number

- Add to the next 10 from a decimal number e.g. $13.6 + 6.4 = 20$
- Add numbers with 2 significant digits only, using mental strategies e.g. $3.4 + 4.8$ e.g. $23\ 000 + 47$

000

- Add 1- or 2-digit multiples of 10, 100, 1000, 10 000 and 100 000 e.g. $8000 + 7000$ e.g. $600\ 000 + 700\ 000$
- Add near multiples of 10, 100, 1000, 10 000 and 100 000 to other numbers e.g. $82\ 472 + 30\ 004$
- Add decimal numbers which are near multiples of 1 or 10, including money e.g. $6.34 + 1.99$ e.g. $\pounds 34.59 + \pounds 19.95$
- Use place value and number facts to add two or more 'friendly' numbers, including money and decimals e.g. $3 + 8 + 6 + 4 + 7$ e.g. $0.6 + 0.7 + 0.4$ e.g. $2056 + 44$

Pupils should be taught to (written methods):

- Use column addition to add two or three whole numbers with up to 5 digits
- Use column addition to add any pair of 2-place decimal numbers, including amounts of money

Year 6

Pupils should be taught to (mental methods):

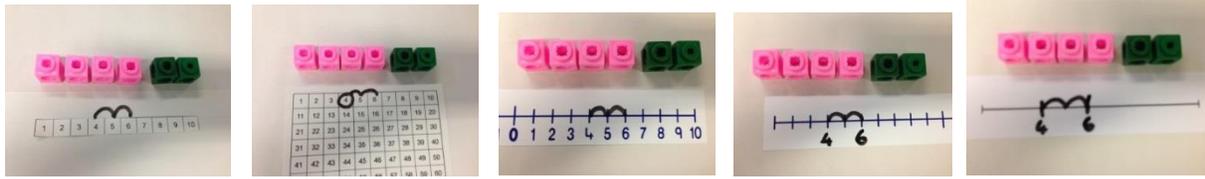
- Know by heart number bonds to 100 and use these to derive related facts e.g. $3.46 + 0.54$
- Derive, quickly and without difficulty, number bonds to 1000
- Add small and large whole numbers where the use of place value or number facts makes the calculation do-able mentally e.g. $34\ 000 + 8000$
- Use rounding to add multiples of 10, 100 or 1000 eg. $6345 + 199$
- Add negative numbers in a context such as temperature where the numbers make sense
- Add two 1-place decimal numbers or two 2-place decimal numbers less than 1 e.g. $4.5 + 6.3$ e.g. $0.74 + 0.33$
- Add positive numbers to negative numbers e.g. Calculate a rise in temperature or continue a sequence beginning with a negative number

Pupils should be taught to (written methods):

- Use column addition to add numbers with up to 5 digits
- Use column addition to add decimal numbers with up to 3 decimal places

STAGE 1

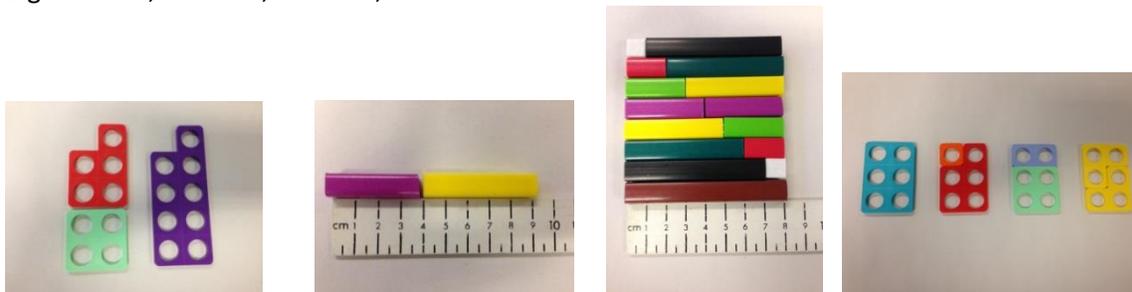
Count on using number tracks / number lines / 100 grids to support.



Develop concept of number bonds, initially to 10 and then to 20.

Record related number facts.

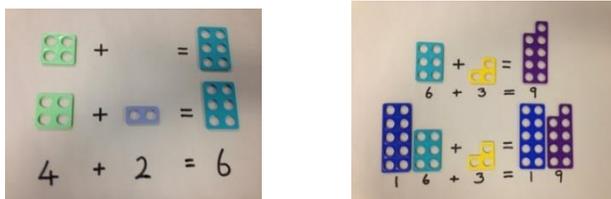
e.g. $4 + 5 = 9$, $5 + 4 = 9$, $9 = 4 + 5$, $9 = 5 + 4$



STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as $4 + \square = 9$



Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $6 + 3 = 9$ (known fact)

$$16 + 3 = 19$$

$$26 + 3 = 29$$

Begin to use understanding of place value and partitioning to carry out addition of one- digit and two-digit numbers. Begin to use a 100 square vertically to add 10s and horizontally to add 1s.



TT 09 -
Number
Square Tool

STAGE 3

TO + O (2 digit number + 1 digit number)

Continue to develop understanding of partitioning and place value and use this to support addition.

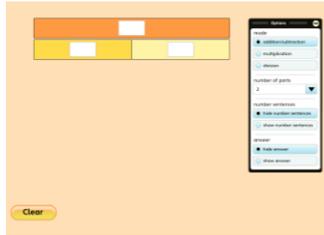
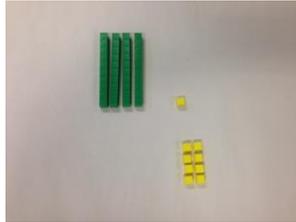
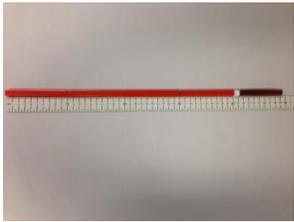
$$41 + 8$$

$$40 + 1 + 8$$

$$40 + 9 = 49$$

Practical apparatus is used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.



TT 39 Bar Modelling tool

When confident with concepts of partitioning and place value, horizontal recording can be replaced with recording in columns with a focus on place value. Resources may need to be used alongside this method initially.

$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 1 \\ + \quad 8 \\ \hline 4 \quad 9 \end{array}$$



TU + O

Continue to develop addition by bridging to the next multiple of 10.

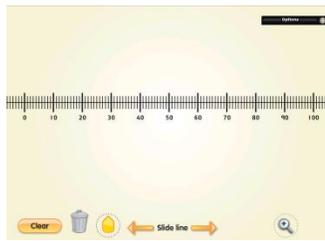
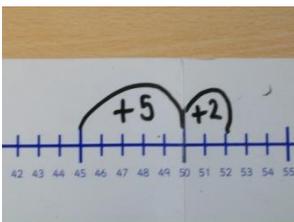
$$45 + 7$$

$$45 + 5 = 50$$

$$50 + 2 = 52$$

Practical apparatus is used to support this, as are number tracks /100 squares and number lines.

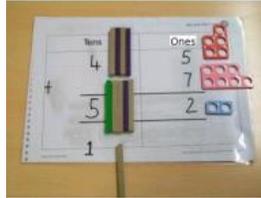
Record the outcomes of calculations in horizontal format.



TT 05 Number line

When confident with concepts of partitioning and place value to bridge 10, horizontal recording can be replaced with recording in columns with a focus on place value. Resources may need to be used alongside this method initially.

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 4 \quad 5 \\
 + \quad 7 \\
 \hline
 5 \quad 2 \\
 \hline
 1
 \end{array}$$



Teach carrying and use expanded method if appropriate (See Stage 5 below)

STAGE 4

TO + TO (2 digit number + 2 digit number)

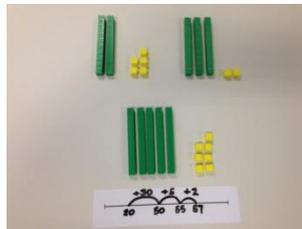
Continue to develop understanding of partitioning and place value and use this to support addition.

$$\begin{aligned}
 25 + 32 \\
 20 + 30 = 50 \\
 5 + 2 = 7 \\
 50 + 7 = 57
 \end{aligned}$$

Practical apparatus is used to support this, as are number tracks /100 squares and number lines. Record the outcomes of calculations in horizontal format.

Where units combine to make totals greater than 10, regroup using partitioning skills

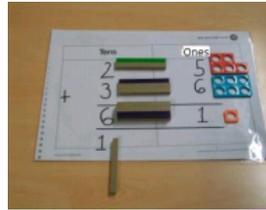
$$\begin{aligned}
 25 + 36 \\
 20 + 30 = 50 \\
 5 + 6 = 11 \\
 50 + 11 = 50 + 10 + 1 = 61
 \end{aligned}$$



Pupils continue to determine when calculations are best carried out using mental strategies.

Horizontal recording can begin to be replaced with recording in columns with a focus on place value. Only use expanded recording and apparatus to illustrate concept initially if required (see stage 5 examples), although the method could be modeled by the teacher otherwise move towards the formal written method initially with resources.

$$\begin{array}{r}
 \text{T O} \\
 25 \\
 + 36 \\
 \hline
 61 \\
 1
 \end{array}$$



STAGE 5

Continue to determine when calculations are best carried out using mental strategies.

When written methods are more appropriate, continue use of practical apparatus to support, develop an understanding of the formal written method for column addition, initially without and then introducing carrying.

Use expanded method to add HTO + HTO (initially without carrying then with carrying)

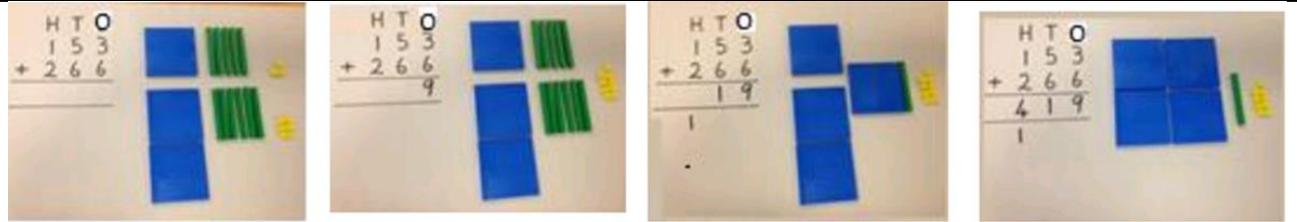
$$\begin{array}{r}
 328 + 436 = \\
 \hline
 \begin{array}{r}
 300 \quad 20 \quad 8 \\
 + 400 \quad 30 \quad 6 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 328 \\
 + 436 \\
 \hline
 \end{array}
 \end{array}$$

STAGE 6

Continue to determine when calculations are best carried out using mental strategies.

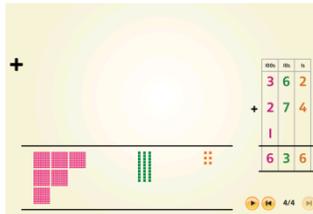
Extend the standard written method to introduce the hundreds column, initially without and then introducing carrying. Only use using expanded recording if appropriate (see below).

$$\begin{array}{r}
 \text{H T O} \\
 153 \\
 + 266 \\
 \hline
 9 \\
 110 \\
 300 \\
 \hline
 419
 \end{array}$$



This becomes

$$\begin{array}{r}
 \text{H T O} \\
 153 \\
 + 266 \\
 \hline
 419 \\
 1
 \end{array}$$



TT 37
Written
calculations tool

The above should be developed to include larger numbers up to 5 digits.

STAGE 7

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to add increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

Develop adding to include adding decimals up to 2 decimal places (Year 5) and up to 3 decimal places (Year 6.)



TT 14
Decimal
Place Value
Cards



TT 10
Decimal
Number
Square Tool

Subtraction Calculation Guidance

Year 1

Pupils should be taught:

- Number bonds ('story' of 5, 6, 7, 8, 9 and 10)
- Count back in 1s from a given 2-digit number
- Subtract one 1-digit number from another
- Count back in 10s from any given 2-digit number
- Subtract 10 from any given 2-digit number
- Use number facts to subtract 1-digit numbers from 2-digit numbers e.g. Use $7 - 2$ to work out $27 - 2$, $37 - 2$

Year 2

Pupils should be taught:

- Number bonds – know all the pairs of numbers which make all the numbers to 12
- Count back in 1s and 10s from any given 2-digit number
- Subtract a 1-digit number from any 2-digit number using number facts, including bridging multiples of 10 e.g. $56 - 3$ e.g. $53 - 5$
- Subtract 10 and small multiples of 10 from any given 2-digit number
- Subtract any pair of 2-digit numbers by counting back in 10s and 1s or by counting up

Year 3

Pupils should be taught to:

- Know pairs with each total to 20 e.g. $22 - 2 = 20$, $17 + 3 = 20$
- Subtract any two 2-digit numbers
- Perform place-value subtractions with ease e.g. $536 - 30 = 506$
- Subtract 2-digit numbers from numbers > 100 by counting up e.g. $143 - 76$ is done by starting at 76. Then add 4 (80), then add 20 (100), then add 43, making the difference a total of 67
- Subtract multiples and near multiples of 10 and 100 e.g. $34 - 9 = 25$, $658 - 99 = 559$
- Subtract, when appropriate, by counting back or taking away, using place value and number facts
- Find change from £1, £5 and £10

Year 4

Pupils should be taught:

- Subtract any two 2-digit numbers
- Know by heart/quickly derive number bonds to 100
- Perform place-value subtractions without a struggle e.g. $4736 - 706 = 4030$

- Subtract multiples and near multiples of 10, 100, 1000, £1 and 10p
- Subtract multiples of 0.1
- Subtract by counting up e.g. $503 - 368$ is done by adding $368 + 2 + 30 + 100 + 3$ (so we added 135)
- Subtract, when appropriate, by counting back or taking away, using place value and number facts
- Subtract £1, 10p, 1p from amounts of money
- Find change from £10, £20 and £50

Year 5

Pupils should be taught to:

- Subtract numbers with 2 significant digits only, using mental strategies e.g. $6.2 - 4.5$
- e.g. $72\ 000 - 47\ 000$
- Subtract 1- or 2-digit multiples of 10, 100, 1000, 10 000 and 100 000 e.g. $8000 - 3000$
- e.g. $60\ 000 - 200\ 000$
- Subtract 1- or 2-digit near multiples of 10, 100, 1000, 10 000 and 100 000 from other numbers e.g. $82\ 472 - 30\ 004$
- Subtract decimal numbers which are near multiples of 1 or 10, including money e.g. $6.34 - 1.99$ e.g. $£34.59 - £19.95$
- Use counting up subtraction, with knowledge of number bonds to 10, 100 or £1, as a strategy to perform mental subtraction e.g. $£10 - £3.45$ e.g. $1000 - 782$
- Recognise fraction complements to 1 and to the next whole number e.g. $1\frac{2}{5} + \frac{3}{5} = 2$.

Year 6

Pupils should be taught to:

- Use number bonds to 100 to perform mental subtraction of any pair of integers by complementary addition e.g. $1000 - 654$ as $46 + 300$ in our heads
- Use number bonds to 1 and 10 to perform mental subtraction of any pair of 1-place or 2-place decimal numbers using complementary addition and including money e.g. $10 - 3.65$ as $0.35 + 6$ e.g. $£50 - £34.29$ as $71p + £15$
- Use number facts and place value to perform mental subtraction of large numbers or decimal numbers with up to 2 places e.g. $467\ 900 - 3005$ e.g. $4.63 - 1.02$
- Subtract multiples of powers of 10 and near multiples of the same
- Subtract negative numbers in a context such as temperature where the numbers make sense

STAGE 1

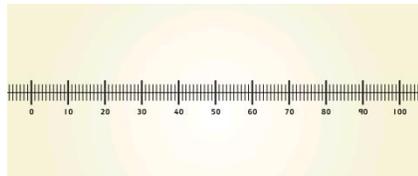
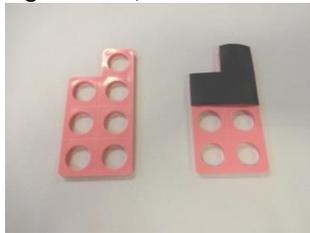
Count back using number tracks / number lines / 100 grids to support the development of the concept of subtraction as take away.



Develop subtraction facts initially to ten and then to 20.

Record related number facts (and make links to related addition facts)

e.g. $9 - 4 = 5$, $9 - 5 = 4$



Number Line tool TT 05

STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as $9 - ? = 5$.

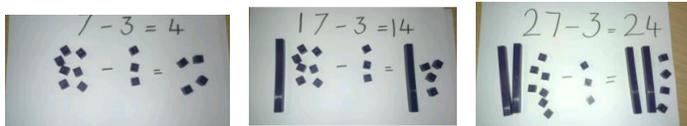
Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $7 - 3 = 4$ (known fact)

TO - O

$17 - 3 = 14$

$27 - 3 = 24$



Begin to use understanding of place value and partitioning to support subtraction of one-digit and two digit numbers

STAGE 3

Practical apparatus are used to support this, as are number tracks /100 squares and number lines. Record the outcomes of calculations in horizontal format.

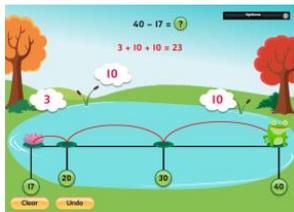
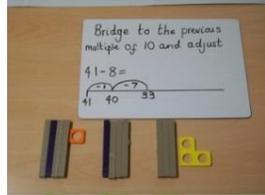
E.g. TO – O (no borrowing) $27 - 6 = 21$

TO - O (Borrowing) Continue to develop subtraction by bridging to the previous multiple of 10 and adjusting. This method builds on mental methods and knowledge of number-bonds.

$$41 - 8 = 33$$

$$41 - 1 = 40$$

$$40 - 7 = 33$$



Use counting up to subtract TO – O and TO – TO e.g. $40 - 17$ count up from 17

Counting up Tool
TT 31

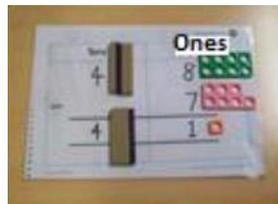
Pupils continue to determine when calculations are best carried out using mental strategies.

STAGE 4

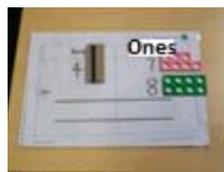
When pupils are confident with the prior stages, horizontal recording can begin to be replaced with recording in columns with a focus on place value. Use expanded recording and apparatus to illustrate the concept initially (if required) before moving towards the formal written method. (See Stage 6 below)

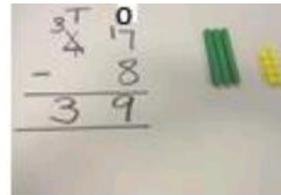
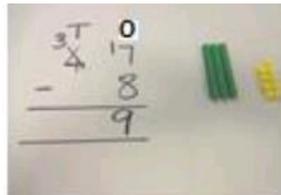
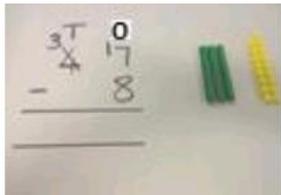
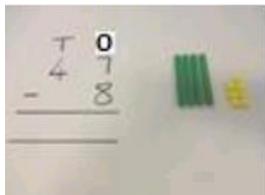
TO – O Using apparatus to support if necessary

$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 8 \\ - \quad 7 \\ \hline 4 \quad 1 \end{array}$$



$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 7 \\ - \quad 8 \\ \hline 4 \quad 1 \end{array} \quad \longrightarrow \quad \begin{array}{r} \text{T} \quad \text{O} \\ 3 \quad 17 \\ - \quad 8 \\ \hline 4 \quad 1 \end{array}$$





STAGE 5

TO - TO

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method. Use expanded recording and apparatus to illustrate concept initially (if required) before moving towards the formal written method (See Stage 6 below)

No exchange

$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 7 \\ - \quad 8 \\ \hline 1 \quad 1 \end{array}$$

Exchange

$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 5 \\ - \quad 2 \quad 6 \end{array} \longrightarrow \begin{array}{r} 40 \quad 5 \\ - \quad 20 \quad 6 \\ \hline 20 \quad 6 \end{array} \quad \begin{array}{r} 30 \quad 15 \\ - \quad 20 \quad 6 \\ \hline 10 \quad 9 = 19 \end{array}$$

Becomes

$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 5 \\ - \quad 2 \quad 6 \end{array} \longrightarrow \begin{array}{r} \text{T} \quad \text{O} \\ 3 \quad 4 \quad 1 \quad 5 \\ - \quad 2 \quad 6 \\ \hline 3 \quad 9 \end{array}$$

STAGE 6

HTO – HTO (3 digit number – 3 digit number)

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method. Only use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

Explore how the process relates to numbers with zeros as place holders.

No exchange

Using an expanded method of recording (if appropriate) before moving to formal method

$$\begin{array}{r}
 \text{H T O} \\
 336 \\
 - 225 \\
 \hline
 \end{array}
 \longrightarrow
 \begin{array}{r}
 300 \quad 30 \quad 6 \\
 - 200 \quad 20 \quad 5 \\
 \hline
 100 \quad 10 \quad 1 = 111
 \end{array}$$

becomes

$$\begin{array}{r}
 \text{H T O} \\
 336 \\
 - 225 \\
 \hline
 111
 \end{array}$$

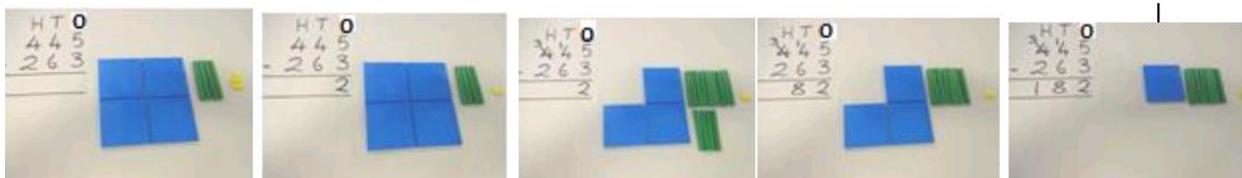
Exchange

$$726 - 243$$

Subtract using expanded method
TT 37

Becomes

Using base 10 445 - 263

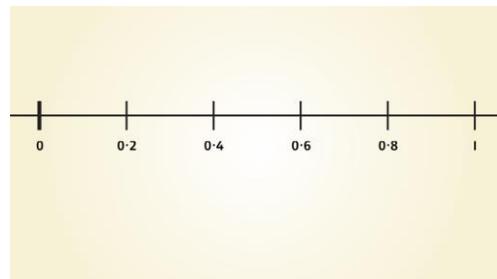


STAGE 7

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to subtraction of increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

Use formal written subtraction to subtract decimal numbers, initially up to 2 decimal places, including money and then up to three decimal places, including money.



Decimal Number line
TT 06

Multiplication Calculation Guidance

Year 1

Pupils should be taught to (Mental Methods):

- Begin to count in 2s, 5s and 10s
- Begin to say what three 5s are by counting in 5s, or what four 2s are by counting in 2s, etc.
- Double numbers to 10

Year 2

Pupils should be taught to (mental methods):

- Count in 2s, 5s and 10s
- Begin to count in 3s
- Begin to understand that multiplication is repeated addition and to use arrays e.g. 3×4 is three rows of 4 dots
- Begin to learn the $\times 2$, $\times 3$, $\times 5$ and $\times 10$ tables, seeing these as 'lots of' e.g. 5 lots of 2, 6 lots of 2, 7 lots of 2
- Double numbers up to 20

- Begin to double multiples of 5 to 100
- Begin to double 2-digit numbers less than 50 with 1s digits of 1, 2, 3, 4 or 5

Year 3

Pupils should be taught to (mental methods):

- Know by heart all the multiplication facts in the $\times 2$, $\times 3$, $\times 4$, $\times 5$, $\times 8$ and $\times 10$ tables
- Multiply whole numbers by 10 and 100
- Recognise that multiplication is commutative (that multiplication calculations can be carried out in any order) e.g. $5 \times 2 = 10$, $2 \times 5 = 10$
- Use place value and number facts in mental multiplication e.g. 30×5 is 15×10
- Partition teen numbers to multiply by a 1-digit number e.g. 3×14 as 3×10 and 3×4 then recombine.
- Double numbers up to 50

Pupils should be taught to (written methods):

- Use partitioning (grid multiplication)
- To multiply 2-digit and 3-digit numbers by 'friendly' 1-digit numbers e.g. 46×5 , 236×4

Year 4

Pupils should be taught to (mental methods):

- Know by heart all the multiplication facts up to 12×12
- Recognise factors up to 12 of 2-digit numbers
- Multiply whole numbers and 1-place decimals by 10, 100, 1000
- Multiply multiples of 10, 100 and 1000 by 1-digit numbers e.g. 300×6 e.g. 4000×8
- Use understanding of place value and number facts in mental multiplication e.g. 36×5 is half of 36×10 e.g. $50 \times 60 = 3000$
- Partition 2-digit numbers to multiply by a 1-digit number mentally e.g. 4×24 as 4×20 and 4×4 then recombine
- Multiply near multiples by rounding e.g. 33×19 as $(33 \times 20) - 33$
- Find doubles to double 100 and beyond using partitioning
- Begin to double amounts of money e.g. $\pounds 35.60$ doubled is $\pounds 71.20$

Pupils should be taught to (written methods):

- Use a vertical written method to multiply a 1-digit number by a 3-digit number (ladder method)
- Use an efficient written method to multiply a 2-digit number by a number between 10 and 20 by partitioning (grid method)

Year 5

Pupils should be taught to (mental methods):

- Know by heart all the multiplication facts up to 12×12
- Multiply whole numbers and 1- and 2-place decimals by 10, 100, 1000, 10 000
- Use knowledge of factors and multiples in multiplication e.g. 43×6 is double 43×3 e.g. 28×50 is half of $28 \times 100 = 1400$
- Use knowledge of place value and rounding in mental multiplication e.g. 67×199 as $67 \times 200 - 67$
- Use doubling and halving as a strategy in mental multiplication e.g. 58×5 is half of 58×10 e.g. 34×4 is 34 doubled twice
- Partition 2-digit numbers, including decimals, to multiply by a 1-digit number mentally e.g. 6×27 as 6×20 (120) plus 6×7 (42) e.g. $6 \cdot 3 \times 7$ as 6×7 (42) plus $0 \cdot 3 \times 7$ (2·1)
- Double amounts of money by partitioning e.g. £37·45 doubled is £37 doubled (£74) plus 45p doubled (90p) giving a total of £74·90

Pupils should be taught to (written methods):

- Use short multiplication to multiply a 1-digit number by a number with up to 4 digits
- Use long multiplication to multiply 3-digit and 4-digit numbers by a number between 11 and 20
- Choose the most efficient method in any given situation
- Find simple percentages of amounts e.g. 10%, 5%, 20%, 15% and 50%
- Begin to multiply fractions and mixed numbers by whole numbers ≤ 10 e.g. $4 \times \frac{2}{3} = \frac{8}{3} = 2 \frac{2}{3}$

Year 6

Pupils should be taught to (mental methods):

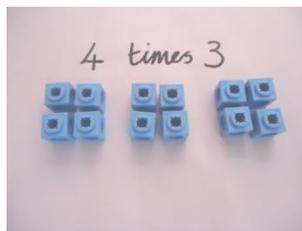
- Know by heart all the multiplication facts up to 12×12
- Multiply whole numbers and decimals with up to 3 places by 10, 100 or 1000 e.g. $234 \times 1000 = 234\ 000$
- e.g. $0 \cdot 23 \times 1000 = 230$
- Identify common factors, common multiples and prime numbers and use factors in mental multiplication
- e.g. 326×6 is 652×3 which is 1956
- Use place value and number facts in mental multiplication e.g. $4000 \times 6 = 24\ 000$ e.g. $0 \cdot 03 \times 6 = 0 \cdot 18$
- Use doubling and halving as mental multiplication strategies, including to multiply by 2, 4, 8, 5, 20, 50 and 25 e.g. 28×25 is a quarter of $28 \times 100 = 700$
- Use rounding in mental multiplication e.g. 34×19 as $(34 \times 20) - 34$
- Multiply 1- and 2-place decimals by numbers up to and including 10 using place value and partitioning e.g. $3 \cdot 6 \times 4$ is $12 + 2 \cdot 4$ e.g. $2 \cdot 53 \times 3$ is $6 + 1 \cdot 5 + 0 \cdot 09$
- Double decimal numbers with up to 2 places using partitioning e.g. $36 \cdot 73$ doubled is double 36 (72) plus double $0 \cdot 73$ (1·46)

Pupils should be taught to (written methods):

- Use short multiplication to multiply a 1-digit number by a number with up to 4 digits
- Use long multiplication to multiply a 2-digit number by a number with up to 4 digits
- Use short multiplication to multiply a 1-digit number by a number with 1 or 2 decimal places, including amounts of money
- Multiply fractions and mixed numbers by whole numbers
- Multiply fractions by proper fractions
- Use percentages for comparison and calculate simple percentages

STAGE 1

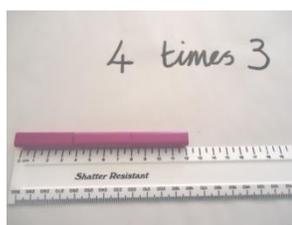
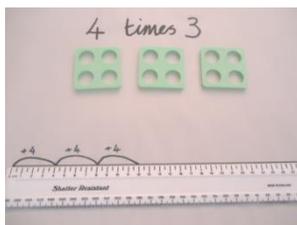
Develop multiplication as repeated grouping (repeated addition of sets of the same size) using practical apparatus and diagrams.



Multiplication and division modeller tool TT 01

STAGE 2

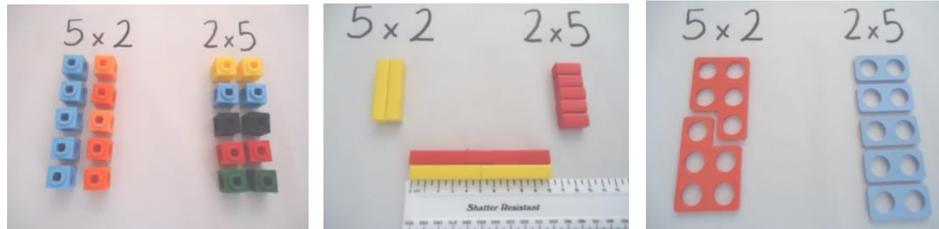
Develop an understanding of multiplication using arrays and number lines, showing repeated groups. Use number lines to show repeated grouping (repeated addition of sets of the same size).



STAGE 3

Develop the use of x and = symbols to record calculations horizontally.

Use arrays and other practical apparatus to illustrate commutativity (that multiplication calculations can be carried out in any order) e.g. 2×5 arrives at the same product as 5×2 .

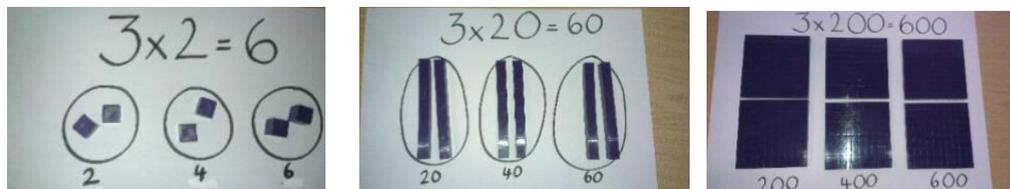


Begin to derive new facts from known facts

e.g. $3 \times 2 = 6$ (known fact)

$30 \times 2 = 60$

$300 \times 2 = 600$ etc



STAGE 4

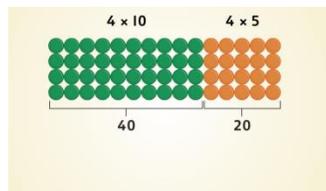
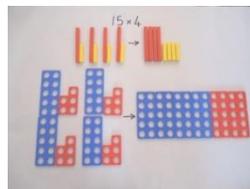
Begin to use understanding of place value and partitioning to carry out multiplication of two-digit by one-digit numbers

$$\begin{array}{r} 15 \times 4 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$

$$10 \times 4 = 40$$

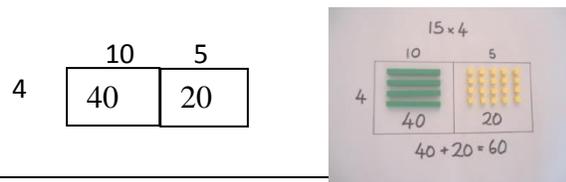
$$5 \times 4 = 20$$

$$40 + 20 = 60$$



Multiplication modeller tool – partitioning TT 02

Use grid approaches to illustrate as appropriate using practical apparatus to support.



Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

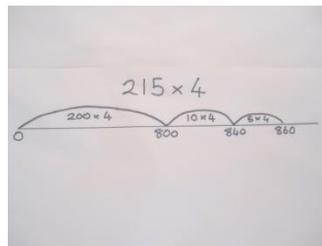
$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 1 \quad 5 \\
 \times \quad \underline{4} \\
 4 \quad 0 \quad (10 \times 4) \\
 \underline{2 \quad 0} \quad (5 \times 4) \\
 6 \quad 0
 \end{array}
 \longrightarrow
 \begin{array}{r}
 \text{T} \quad \text{O} \\
 1 \quad 5 \\
 \times \quad \underline{4} \\
 \underline{6 \quad 0} \\
 2
 \end{array}$$

STAGE 5

Extend written approaches to HTO x O, then to Th,HTO x O
Illustrate using partitioning approaches as required.

$$\begin{array}{c}
 215 \times 4 \\
 \swarrow \quad \downarrow \quad \searrow \\
 200 \quad 10 \quad 5
 \end{array}$$

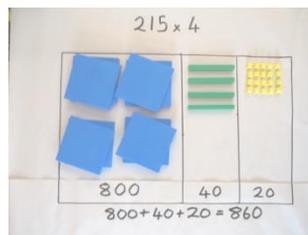
$$\begin{aligned}
 200 \times 4 &= 800 \\
 10 \times 4 &= 40 \\
 5 \times 4 &= 20 \\
 800 + 40 + 20 &= 860
 \end{aligned}$$



Illustrate using grid approaches as required

$$\begin{array}{r}
 200 \quad 10 \quad 5 \\
 4 \quad \boxed{\begin{array}{|c|c|c|} \hline 800 & 40 & 20 \\ \hline \end{array}}
 \end{array}$$

$$800 + 40 + 20 = 860$$



Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{O} \\
 1 \quad 1 \quad 5 \\
 \times \quad \underline{4} \\
 \quad \quad 2 \quad 0 \quad (5 \times 4) \\
 \quad \quad 4 \quad 0 \quad (10 \times 4) \\
 \underline{8 \quad 0 \quad 0} \quad (200 \times 4) \\
 8 \quad 6
 \end{array}
 \longrightarrow
 \begin{array}{r}
 \text{H} \quad \text{T} \quad \text{O} \\
 2 \quad 1 \quad 5 \\
 \times \quad \underline{4} \\
 \underline{8 \quad 6 \quad 0} \\
 2
 \end{array}$$

STAGE 6

Extend written approaches to HTO x TO and Th, HTO x TO Illustrate using grid approaches as required

26 x 13

	20	6	
10	200	60	
3	60	18	

$$200 + 60 + 60 + 18 = 338$$

Develop expanded recording in columns and then move to formal written method of long multiplication, using practical apparatus to support as required.

<table style="border-collapse: collapse;"> <tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr> <tr><td></td><td></td><td>2</td><td>6</td><td></td></tr> <tr><td>x</td><td></td><td>1</td><td>3</td><td></td></tr> <tr><td></td><td></td><td>1</td><td>8</td><td>(6 x 3)</td></tr> <tr><td></td><td></td><td>6</td><td>0</td><td>(20 x 3)</td></tr> <tr><td></td><td></td><td>6</td><td>0</td><td>(6 x 10)</td></tr> <tr><td></td><td>2</td><td>0</td><td>0</td><td>(10 x 20)</td></tr> <tr><td></td><td>3</td><td>3</td><td>8</td><td></td></tr> <tr><td></td><td>1</td><td></td><td></td><td></td></tr> </table>		H	T	O				2	6		x		1	3				1	8	(6 x 3)			6	0	(20 x 3)			6	0	(6 x 10)		2	0	0	(10 x 20)		3	3	8			1				→	<table style="border-collapse: collapse;"> <tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr> <tr><td></td><td></td><td>2</td><td>6</td><td></td></tr> <tr><td>x</td><td></td><td>1</td><td>3</td><td></td></tr> <tr><td></td><td></td><td>7</td><td>8</td><td></td></tr> <tr><td></td><td></td><td>1</td><td></td><td></td></tr> <tr><td></td><td>2</td><td>6</td><td>0</td><td></td></tr> <tr><td></td><td>3</td><td>3</td><td>8</td><td></td></tr> <tr><td></td><td>1</td><td></td><td></td><td></td></tr> </table>		H	T	O				2	6		x		1	3				7	8				1				2	6	0			3	3	8			1			
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226 x 13

	200	20	6	
10	2000	200	60	
3	600	60	18	

$$2000 + 600 + 200 + 60 + 60 + 18 = 2938$$

Develop expanded recording in columns and then move to formal method of long multiplication, using practical apparatus to support as required.

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Division Calculation Guidance

Year 1

Pupils should be taught to (mental methods):

- Begin to count in 2s, 5s and 10s
- Find half of even numbers to 12 and know it is hard to halve odd numbers
- Find half of even numbers by sharing
- Begin to use visual and concrete arrays or 'sets of' to find how many sets of a small number make a larger number

Year 2

Pupils should be taught to (mental methods):

- Count in 2s, 5s and 10s
- Begin to count in 3s
- Using fingers, say where a given number is in the 2s, 5s or 10s count e.g. 8 is the fourth number when I count in 2s
- Relate division to grouping e.g. How many groups of 5 in 15?
- Halve numbers to 20
- Begin to halve numbers to 40 and multiples of 10 to 100
- Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{3}{4}$ of a quantity of objects and of amounts (whole number answers)

Year 3

Pupils should be taught to (mental methods):

- Know by heart all the division facts derived from the $\times 2$, $\times 3$, $\times 4$, $\times 5$, $\times 8$ and $\times 10$ tables
- Divide whole numbers by 10 or 100 to give whole number answers
- Recognise that division is not commutative
- Use place value and number facts in mental division e.g. $84 \div 4$ is half of 42
- Divide larger numbers mentally by subtracting the 10th multiple as appropriate, including those with remainders e.g. $57 \div 3$ is $10 + 9$ as $10 \times 3 = 30$ and $9 \times 3 = 27$
- Halve even numbers to 100, halve odd numbers to 20

Pupils should be taught to (written methods):

- Perform divisions just above the 10th multiple using horizontal or vertical jottings and understanding how to give a remainder as a whole number

- Find unit fractions of quantities and begin to find non-unit fractions of quantities

Year 4

Pupils should be taught to (mental methods):

- Know by heart all the division facts up to $144 \div 12$
- Divide whole numbers by 10, 100, to give whole number answers or answers with 1 decimal place
- Divide multiples of 100 by 1-digit numbers using division facts e.g. $3200 \div 8 = 400$
- Use place value and number facts in mental division e.g. $245 \div 20$ is half of $245 \div 10$
- Divide larger numbers mentally by subtracting the 10th or 20th multiple as appropriate e.g. $156 \div 6$ is $20 + 6$ as $20 \times 6 = 120$ and $6 \times 6 = 36$
- Find halves of even numbers to 200 and beyond using partitioning
- Begin to halve amounts of money e.g. half of $\pounds 52.40$ is $\pounds 26.20$

Pupils should be taught to (written methods):

- Use a written method to divide a 2-digit or a 3-digit number by a 1-digit number
- Give remainders as whole numbers
- Begin to reduce fractions to their simplest forms
- Find unit and non-unit fractions of larger amounts

Year 5

Pupils should be taught to (mental methods):

- Know by heart all the division facts up to $144 \div 12$
- Divide whole numbers by 10, 100, 1000, 10 000 to give whole number answers or answers with 1, 2 or 3 decimal places
- Use doubling and halving as mental division strategies e.g. $34 \div 5$ is $(34 \div 10) \times 2$
- Use knowledge of multiples and factors, as well as tests for divisibility, in mental division e.g. $246 \div 6$ is $123 \div 3$ e.g. We know that 525 divides by 25 and by 3
- Halve amounts of money by partitioning e.g. $1/2$ of $\pounds 75.40 = 1/2$ of $\pounds 75$ ($\pounds 37.50$) plus half of 40p (20p) which is $\pounds 37.70$
- Divide larger numbers mentally by subtracting the 10th or 100th multiple as appropriate e.g. $96 \div 6$ is $10 + 6$, as $10 \times 6 = 60$ and $6 \times 6 = 36$ e.g. $312 \div 3$ is $100 + 4$ as $100 \times 3 = 300$ and $4 \times 3 = 12$
- Know tests for divisibility by 2, 3, 4, 5, 6, 9 and 25
- Know square numbers and cube numbers
- Reduce fractions to their simplest form

Pupils should be taught to (written methods):

- Use short division to divide a number with up to 4 digits by a number ≤ 12
- Give remainders as whole numbers or as fractions e.g. $457 \div 2 = 228 \text{ r } 1$ or $228 \frac{1}{2}$
- Find non-unit fractions of large amounts
- Turn improper fractions into mixed numbers and vice versa
- Choose the most efficient method in any given situation

Year 6

Pupils should be taught to (mental methods):

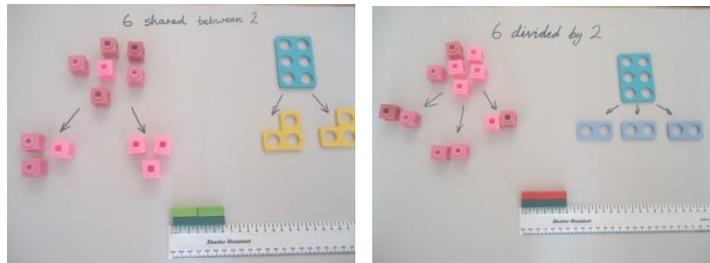
- Know by heart all the division facts up to $144 \div 12$
- Divide whole numbers by powers of 10 to give whole number answers or answers with up to 3 decimal places
- Identify common factors, common multiples and primes numbers and use factors in mental division e.g. $438 \div 6$ is $219 \div 3$ which is 73
- Use tests for divisibility to aid mental calculation
- Use doubling and halving as mental division strategies, for example to divide by 2, 4, 8, 5, 20 and 25 e.g. $628 \div 8$ is halved three times: 314, 157, 78.5
- Divide 1- and 2-place decimals by numbers up to and including 10 using place value e.g. $2.4 \div 6 = 0.4$ e.g. $0.65 \div 5 = 0.13$ e.g. $\pounds 6.33 \div 3 = \pounds 2.11$
- Halve decimal numbers with up to 2 places using partitioning e.g. Half of 36.86 is half of 36 (18) plus half of 0.86 (0.43)
- Know and use equivalence between simple fractions, decimals and percentages, including in different contexts
- Recognise a given ratio and reduce a given ratio to its lowest terms

Pupils should be taught to (written methods):

- Use short division to divide a number with up to 4 digits by a 1-digit or a 2-digit number
- Use long division to divide 3-digit and 4-digit numbers by 'friendly' 2-digit numbers
- Give remainders as whole numbers or as fractions or as decimals
- Divide a 1-place or a 2-place decimal number by a number ≤ 12 using multiples of the divisors
- Divide proper fractions by whole numbers

STAGE 1

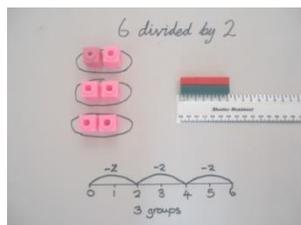
Develop division as sharing



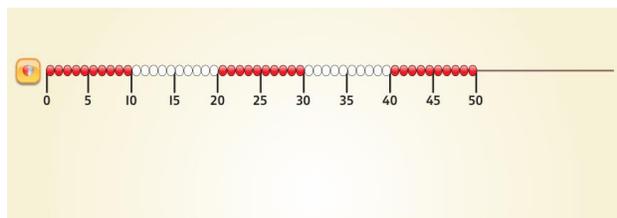
Note vocabulary: 6 shared by two means 'share 6 into two equal parts'
6 divided by 2 means 'how many 2's are in 6?'

STAGE 2

Develop an understanding of division using arrays and number lines showing repeated groups
Use number lines to show repeated grouping (repeated subtraction of sets of the same size)



Use a bead string to count up in repeated steps eg $50 \div 5 = 10$



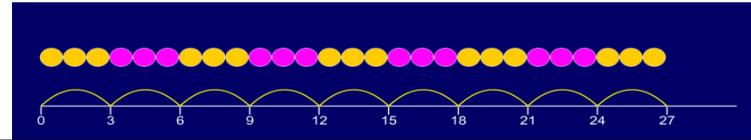
Bead string tool
TT 03

STAGE 3

Develop the use of \div and $=$ symbols to record calculations horizontally
Use arrays and other practical apparatus to illustrate making of repeated groups
Begin to derive new facts from known facts
e.g. $6 \div 2 = 3$ (known fact)
 $60 \div 2 = 30$
 $600 \div 2 = 300$

Begin to carry out division of two-digit by one-digit numbers, first without remainders, then introducing remainders, illustrating this using informal methods first if required.

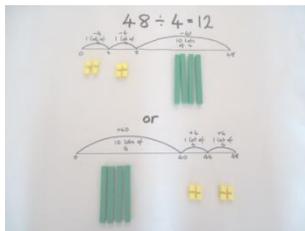
$$27 \div 3$$



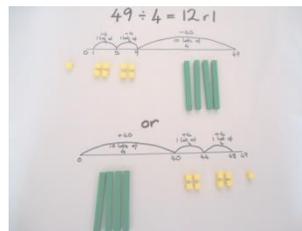
STAGE 4

Division using larger multiples of the divisor, first with no remainders, then with remainders

$$48 \div 4$$



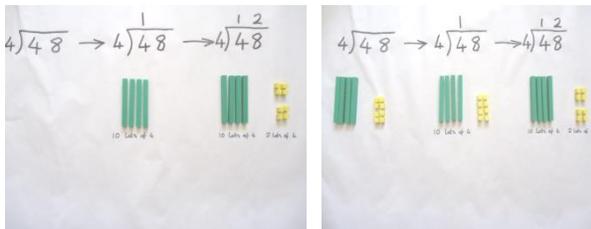
$$49 \div 4$$



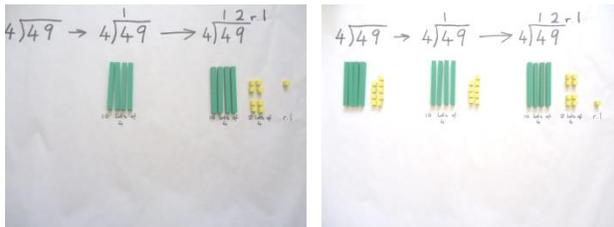
STAGE 5

Move to develop the standard method for short division, first with no remainders, then with remainders
'Bus Stop method' eg $49 \div 4 = 12 \text{ r } 1$

$$48 \div 4$$



$$49 \div 4$$



STAGE 6

Extend written calculation methods to $\text{HTO} \div \text{O}$, then to $\text{Th,HTO} \div \text{O}$, first with no remainders, then with remainders, illustrating this using informal methods first if required.

No carrying forward required

$$448 \div 4$$

(as above, but with additional hundreds column)

No carrying forward required, but with remainders

$$449 \div 4$$

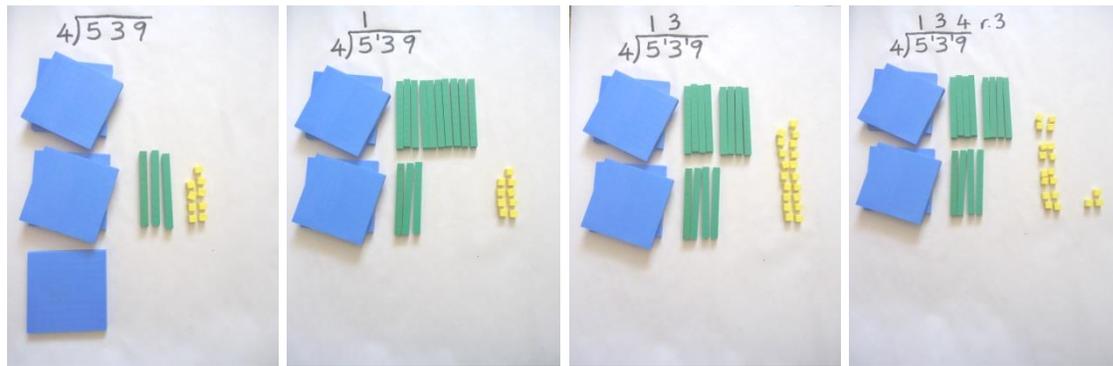
(as above, but with additional hundreds column)

Carrying forward required

$$536 \div 4$$

Carrying forward required, but with remainders

$$539 \div 4$$



Include: fractions as remainders and as decimals

e.g. $57 \div 4 = 14 \text{ r } 1$ (which is $14 \frac{1}{4}$)

$$\begin{array}{r} 14 \quad 2 \quad 5 \\ 4 \overline{) 57} \quad . \quad 0 \quad 0 \\ \underline{40} \quad \quad 1 \quad 1 \quad 2 \\ \quad 17 \quad \quad \quad 1 \quad 2 \\ \quad \underline{16} \quad \quad \quad \quad 2 \\ \quad \quad 1 \quad \quad \quad \quad 2 \\ \quad \quad \underline{1} \quad \quad \quad \quad 0 \\ \quad \quad \quad 0 \quad \quad \quad \quad 0 \end{array}$$

STAGE 7

Extend written approaches to the formal method of long division when dividing by two-digit numbers.

$$\begin{array}{r} 200 + 30 + 3 \\ 24 \overline{) 5600} \\ \underline{4800} \\ 800 \\ \underline{720} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

$$1 \times 24 = 24$$

$$2 \times 24 = 48$$

$$3 \times 24 = 72$$

$$4 \times 24 = 96$$

$$5 \times 24 = 120$$

$$\text{Answer} = 233 \frac{1}{3}$$

